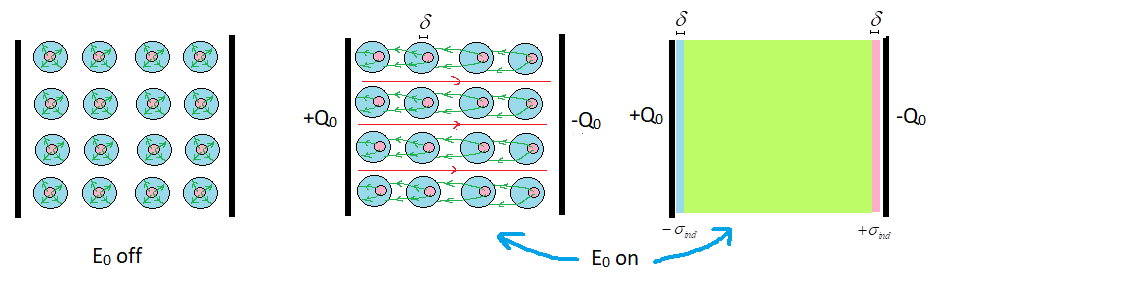
**Classical Model of Dielectric and Dimagnetic Materials**

So we can calculate the electric and magnetic fields when we know what all the charges and currents are. For this reason, we run into a problem when we try to calculate the electric and magnetic fields in the presence of generic materials such as insulators and metals. This is because, for instance, if we place a charge near an insulator, the electric field due to this charge will cause the charges in the insulator to displace slightly – pulling all the positively charged nuclei towards it, and the orbiting electrons away from it. Due to the displacement of these internal charges, we will have a non-zero net charge density inside the insulator when before, we had none. And so the insulator will generate an electric field itself, an induced electric field. And we’d like to calculate it, but we can’t, unless we know what the aforementioned displacement is. Similar arguments apply to magnetic fields. If we place a currnent loop, generating a magnetic field, next to the insulator, the magnetic field will induce the electron clouds of the atoms to spin (or reorient their former spin) too, creating their own magnetic fields. And we need to know by how much their spin changes, to know what their (induced) magnetic field will be. So we’ll look into this here. Just want to try to propose a simple classical, non-statistical (meaning we’ll presume it’s in just one state and so has only one response), model of the susceptibilities.

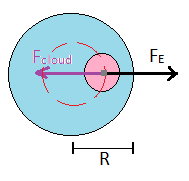
**Simple model for the dielectric**

Consider a dielectric placed between two plates again. We can model the atoms in the material as a positive nucleus, charge q, centered about a spherical electronic cloud of charge q.



For the sake of discussion we’ll assume there are n atoms per unit volume. The electric field between the plates, due to the charge Q0/-Q0 on each side, will polarize the molecules, displacing the electronic cloud a distance δ. We can ultimately think of the sum of displaced clouds to the left, and the absence of such to the right, as a net surface charge density on either end.

So first let’s get the displacement. It’s easier to pretend te clouds are fixed, and the nuclei mobile. Let’s just apply N2L to the nuclei.



There are two forces acting on the nucleius, one the ambient electric field (due to all charges really but leaving out the one from the immediate cloud), now reduced, which we’ll call E, and the other is the field due to the aforementioned immediate cloud. The field due to the cloud is, from Gauss’s law:



and so the two forces are:



So the displacement is proportional to the field. The dipole moment is therefore p = qδ = qR3E/kq = R3E/k. The dipole moment per unit volume would be P = np = nR3E/k = 4πε0nR3E. Considering our formal relationship between P and E, we can write:



Might note an alternative way to write it:



where Vatom is V/N where V is total volume of dielectric and N the number of atoms in the dielectric – so it’s the average molecular volume. In this format, it would appear that we should get χ ~ 3. Anyway, so then we want to relate the bound field to the free field. One way is to get the bound surface charge. This is just P∙n, where n is the unit normal vector to the surface, which is σ = nR3E/k. But now we need E. A simple way to relate the old field to the new one is to view it this way. The amount of separated charge on either end is this: Qinduced = q(nAδ). Filling in our result for E we have:



Note that we can therefore say the induced charge density along the edge will be:



where χ is called the electric susceptibility. So the new field, E, is the old field + the new one created by these separated charges. And so the total field is:



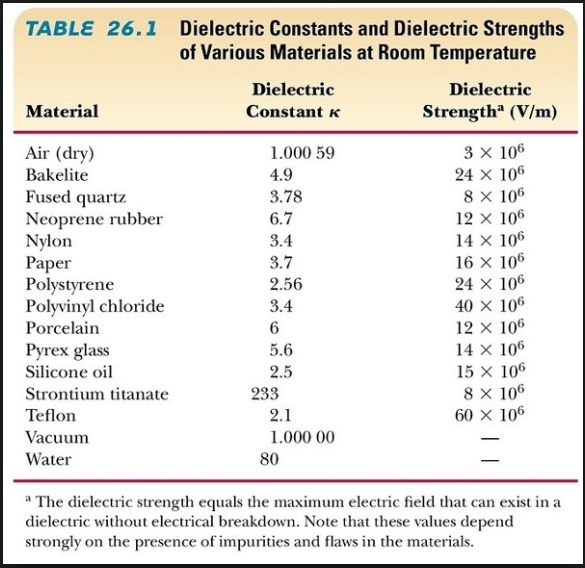
Solving for E we get:



which we can write as:



where χ is called the electric susceptibility. Given our result for χ above, this would make κ ~ 4 for the most part. And this is the correct order of magnitude for most materials, as can see below:



Finally can write σinduced in terms of σ. We have:



and so,



(well negative of that actually). Note that a metal can be treated as having a large cloud, i.e. R → ∞, in which case the susceptibility, χ → ∞, and σinduced → σ, and E → 0 – all well known results for metals. One more miscellaneous result…the polarization per unit volume of the dielectric can be written as P = σinduced (from equation above), and so it would be given by (χ/κ)σ, of course.

**Simple model for a dimagnetic?**

So the dielectric model didn’t require any thermodynamics or quantum mechanics, but dimagnetic models kind of do (though we can sort of get around this for a diamagnet model). According to quantum mechanics, the Hamiltonian of a dipole in a magnetic field is:



where the magnetic moment operator is given by, in the symmetric gauge:



where **S**T the (total) spin angular momentum operator, and **L**T(phys) is the physical angular momentum operator, which, in the symmetric gauge, **A** = -(1/2)**r**×**B**, comes to:



where r⊥ is the component of the position operator perpendicular to the field. The T subscript refers to summing over all electrons in a typical atom. So M is:



And so H comes to, for what it’s worth.

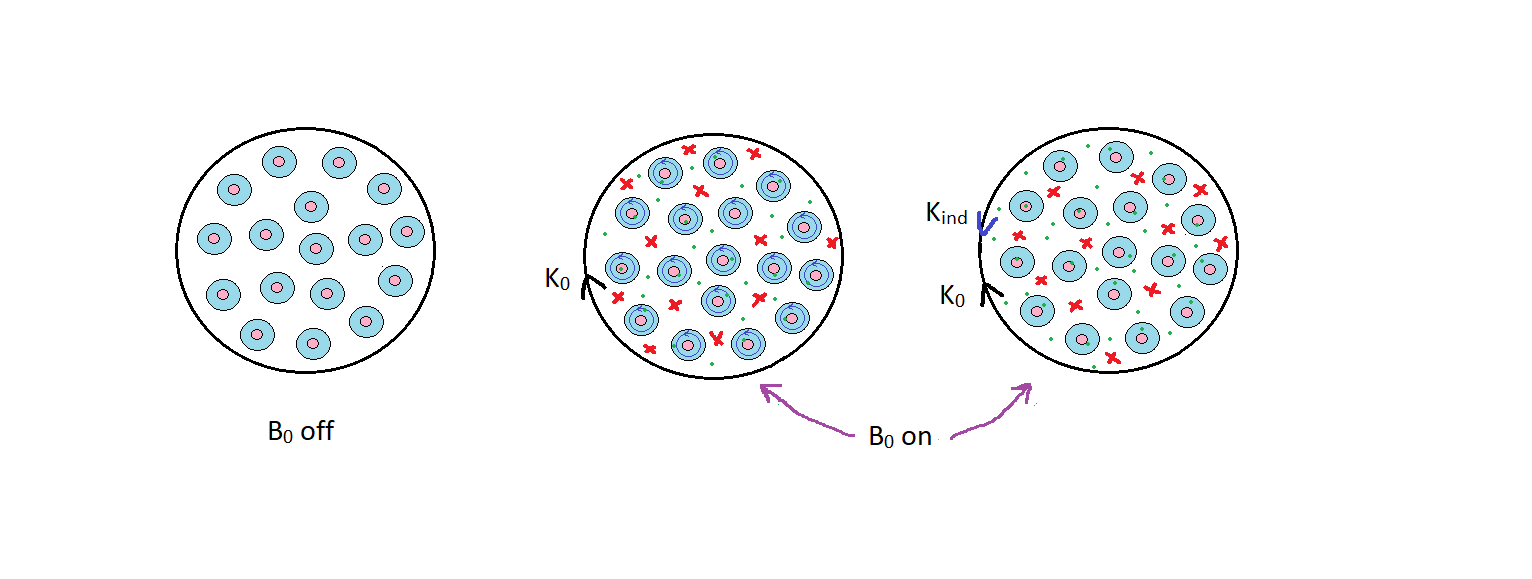


The B term quantifies the paramagnetic response of the atom to the field, and the B2 term quantifies the diamagnetic response, in some sense. For metals, the outer shell electrons are not confined by an appreciably strong Hatom, and so due to how large r⊥ can be, that tends to make the diamagnetic response predominate. For insulators, the reverse is true and the paramagnetic response is largest (unless it should happen, like in carbon, that the average L + gS value is zero for the electrons, in which case only the diamagnetic response would be left).

**Diamagnet Model**

So like we said above, we can construct a classical model for a diamagnet. If we have, say, an atom with no net angular momentum, so that equal numbers of electrons are going CW as CCW, and we turn on a magnetic field in the z direction, then by Faraday’s law a clockwise induced emf will be generated, which will speed up the CCW electrons and slow down the CW ones (force goes in opposite direction to field). And so we will then have a net flow of electrons CCW, which will correspond to a net current CW, and so a net magnet moment in the -z direction, i.e., opposite to the B field.

So let’s be more quantitative. Consider a diamagnetic placed in between a solenoid. This is a top-down view. The solenoid is the outer cylinder (infinite length, technically). And I’ll model the atoms in the material as tiny radius R cylinders (also infinite length, technically, to make calculation easier) with a positive cylindrical nucleic charge q, centered about a cylindrical electronic cloud of charge -q. I’ll take these atoms to be a sort of metallic entity so that the charge cloud can sort of flow around the nucleus w/o resistance. And actually, even though I fill in the interior of the little atomic cylinders, I’m going to model the electron cloud as just a cylindrical *surface* of radius R, to make it easier.



Let’s consider that we start a current If (I0) per wire flowing around the exterior, ultimately constituting a surface current Kf (K0) around the perimeter of the ‘solenoid’. It will generate a magnetic field, B0 = μ0(N/L)If = μ0I0/L = μ0K0, permeating the interior (red x’s). B0 is basically our h above, but with the proper magnetic field units. Starting up this field will generate an induced emf around each of the little molecules, which will induce a current around each of the molecules (which we could think of in turn as constituting a net induced surface current), which will in turn generate molecular magnetic fields (green dots). And these induced magnetic fields will each influence each other’s induced currents by generating their own fluxes inside each molecule. First, like how we got a formula for the polarization of our dielectric before, I’d like to work out a formula for the magnetization of our substance here. This requires knowing what current is induced around each molecule as we raise the free current from 0 to If, and likewise, the field from 0 to B0. So let’s consider a particular molecule, and its induced field Bcloud. Let B be the field generated by the free current and N-1 other induced atomic currents. This is analogous to how E was defined the same way, basically, in the previous problem. We can work this out with N2L on a chosen molecule. We’ll be using Faraday’s law E = (1/2πR)·dΦB/dt, and Bsolenoid = μ0nI = μ0NI/L μ0Icloud/L = μ0K (where K is the surface (side of the cylinder) current density). Will also use fact that current I = λv = (q/2πR)v.



So we may take:



We can then relate the interior magnetization as a whole to the field. We have M = nMcloud,



M is supposed to be of the form χm/(1+χm) ∙ B/μ0, which isn’t *quite* like this. So, what is χm? Let a be the numerator (sans negative sign) in that fraction relating M and B/μ0. Then we need:



So



Or it might be better to write this as:



b could go from 0 to ∞, so χm would have a range of (-1/2,0) as befits a diamagnet. So we can say,



Next we’d like to relate the bound current field to the free current field. So we’ll get the net bound current. This is:



And so the bound field is given by:



And so the net field, in terms of the free field, is:



So we have:



Can also use these expressions to relate the bound current to the free current, Bb = χ­m/(1+χm)B = χm/(1+χm)∙κmB0 = χmB0. And so Kb = χmK0.

**What about a Paramagnet/Ferromagnet?**

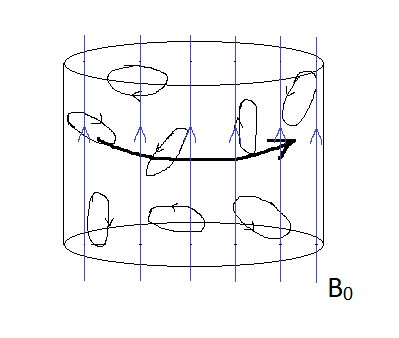
Going back to,



Supposing r⊥ is smallish, then in the small B limit, this reduces to:



Classically, this energy term looks like what we’d get for a constant current dipole rotating in a constant magnetic field (see Physics 2).



For instance, the work done by the field on such a dipole would be:



where we might recall the vector product rule **A**·(**B**×**C**) = **B**·(**C**×**A**) = **C**·(**A**×**B**), and how if a vector of constant magnitude, **Q**, rotates by δ**θ**, then it changes by δ**θ**×**Q** (see Classical Mechanics, accelerated reference frames, e.g.). And so taking the negative integral of this we get:



So this classical interpretation captures some qualitative details, like how if we increase the field, it will exert a torque on the magnetic moment, and cause it to tend to align with the field.

But problem is in the quantitative ones. Such a model wouldn’t account for the observed linear proportionality between **B** and the overall magnetization **M**. So with no field present, we’d expect all the magnetic moments to be randomly oriented and so vectorally add up to zero initially. Once **B** is introduced, the current loops will prefer to line up with **B**, to lower their energy. If prefer, **B** will exert a torque to line the dipoles up. But, it would seem that all current loops should just immediately jump into that orientation. And so the susceptibility would basically be infinite, since as B: 0 → 0+, we get **M**: 0 → finite. Another problem is that the overall magnetic moment of the dipoles should decrease as they line up with the field, thanks to the back-emf induced by the ambient magnetic field exerting forces along the perimeter of the loop (or tangent to the dipole current basically). Yet in paramagnetic/ferromagnetic systems, the magnetic moment has a constant magnitude rather, at least for low lying states. A related problem is that classically, the magnetic field cannot do any actual work on these dipoles. It does positive work by causing the dipoles to align with the field, only at the expense of equal-and-opposite back-emf work against the current/dipole moment. Yet quantum mechanically, we do find that the energy of the dipoles unequivocally changes (can see by release of photons, say, between states with different dipole moment components). Well, we can imagine that the current in the atomic loop is just magically constant somehow, but then we’re back to the infinite susceptibility problem. One way around this is to say that the loops are not initially stationary, but are themselves rotating in random orientations – basically we’re introducing thermal fluctuations. But problem is that when introduce the field, it would seem that should just induce the dipoles to precess about the field, rather than line up with it (see the dynamics file previous)? So we can’t really model this w/o quantum mechanics, and thermodynamics, it seems. So simplest model, for calculation purposes is found in Thermodynamics file (ultimately derived in Stat Mech file). We have for the energy and entropy of such a paramagnet:



where B is field and T is temperature. And consequently, magnetization given by:



Could note that this evinces we can write E = E0 – MB.